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Chiral SU(N) gauge theories and the Konishi anomaly

Riccardo Argurio, Gabriele Ferretti and Rainer Heise

Institute for Theoretical Physics - Göteborg University, and
Chalmers University of Technology, 412 96 Göteborg, Sweden
E-mail: ???, ferretti@fy.chalmers.se, ???

Abstract: We study chiral SU(N) supersymmetric gauge theories with matter in the antifundamental and antisymmetric representations. For SU(5) with two families, we show how to reproduce the non-perturbatively generated superpotential, and we discuss dynamical supersymmetry breaking purely in terms of the Konishi anomaly. We apply the same technique in general to SU(N) with one family. We also briefly comment on the chiral ring for these theories.

Keywords: Nonperturbative Effects, Anomalies in Field and String Theories, Supersymmetry Breaking, Supersymmetric Effective Theories.

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1. Introduction

In the context of $\mathcal{N} = 1$ supersymmetric gauge theories, a very interesting possibility is to break supersymmetry by non-perturbative dynamics. However, dynamical supersymmetry breaking is only possible in a restricted class of theories, the so-called chiral theories, i.e. those with a matter content which does not allow to write a mass term for all matter fields. Chiral theories with gauge group SU(N) are the most well-known examples of theories where dynamical supersymmetry breaking can occur.

That dynamical supersymmetry breaking does occur in a number of chiral theories has been convincingly argued some time ago [1, 2]. The two lines of arguments put forward are the following. One can determine the non-perturbatively generated superpotential by symmetry arguments, and show that together with the tree-level superpotential it does not lead to an extremum [1]. Alternatively, one can use the Konishi anomaly relations [3] to prove that in a supersymmetric vacuum the gluino condensate $S = -\frac{1}{32\pi^2}\,\mathrm{tr}\, W^\alpha W_\alpha$ must vanish, and then show with a one-instanton calculation that S is actually non-zero in the vacuum [2], thus contradicting the assumption of a supersymmetric vacuum.

The object of the present paper is to discuss dynamical supersymmetry breaking by using the Konishi anomaly to determine the non-perturbative piece of the effective superpotential. If such a term is non-zero, one can then see whether the addition of a tree level superpotential leads to a supersymmetric vacuum or not. If, on the other hand, there is an insufficient number of gauge invariants, the Konishi relations typically lead to a vanishing gluino condensate S. One then has to resort to the arguments of [1, 2] for the case where no non-perturbative superpotential is expected, which point towards a strongly coupled non-supersymmetric vacuum.

Although the introduction and use of the generalized Konishi anomaly [4] has recently led to some new insight in the dynamics of $\mathcal{N} = 1$ gauge theories, we only make use of the original Konishi anomaly. In one instance we can use a slight generalization, which

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leads however only to a classical relation (as in [5] for SQCD with baryonic deformations). We use a simplified version of the method outlined in [6], where it was applied to one particular chiral theory (which does not have dynamical supersymmetry breaking). For a related discussion see [7].

The generalization of the Konishi anomaly seems non-trivial for these theories which do not have a matter field in the adjoint like in [4] (or at least in a real representation with two indices, see [8]). Similarly, for the same reason (and because of the impossibility to write mass terms for the matter fields) it seems difficult to have a matrix model formulation of the chiral theories in the spirit of [9, 10].

In the following, we will consider first of all the SU(5) gauge theory with 2 families of chiral matter in the $\mathbf{10}$ and $\bar{\mathbf{5}}$ representations, which is the most celebrated chiral theory where dynamical supersymmetry breaking occurs at arbitrarily small coupling. We then proceed to consider more generally the SU(N) theories with one antisymmetric and $N-4$ antifundamental (i.e. one family), which can have a supersymmetric vacuum for N even while they break supersymmetry dynamically (but at strong coupling) for N odd. Finally we show that for N odd and one family the chiral ring is generated only by the glueball superfield and by the basic invariants.

2. SU(5) chiral gauge theory with two families

The matter content of this SU(5) gauge theory is given by 2 flavors of anti-fundamentals $\tilde{F}_a^{\tilde{i}}$ and 2 flavors of antisymmetric tensors T_i^{ab} [1, 2]. The matter content is of course such that the gauge anomaly cancels. The global symmetries of the theory are at the classical level $\mathrm{SU}(2)_{\tilde{F}} \times \mathrm{SU}(2)_T \times \mathrm{U}(1)_{\tilde{F}} \times \mathrm{U}(1)_T \times \mathrm{U}(1)_R$. At the quantum level, the U(1)s are anomalous, but 2 anomaly-free combinations can always be found. We refrain from proceeding further on the analysis of the U(1) charges since the Konishi anomaly approach is precisely a systematic way to obtain the results of that analysis.

There are 6 independent gauge invariants one can build from the matter fields:

$$X_i^{\tilde{i}} = \tilde{F}_a^{\tilde{i}} \epsilon_{bcdef} T_j^{ab} T_k^{cd} T_i^{ef} \epsilon^{jk} \,, \tag{2.1}$$

$$Y_i = \epsilon_{\tilde{i}\tilde{j}} \tilde{F}_a^{\tilde{i}} \tilde{F}_b^{\tilde{j}} T_i^{ab} \,, \tag{2.2}$$

where ϵ_{bcdef}, $\epsilon_{\tilde{i}\tilde{j}}$ and ϵ^{jk} are the Levi-Civita invariant tensors of SU(5), $\mathrm{SU}(2)_{\tilde{F}}$ and $\mathrm{SU}(2)_T$ respectively.

The above 6 invariants parameterize the 6 dimensional moduli space of the theory. In a generic point on the moduli space the gauge group is totally broken, consistently with the counting that the 24 gauge bosons eat up 24 out of the 30 matter fields, leaving 6 massless singlets representing the classical flat directions which satisfy the D-flatness conditions (see [11] for a complete parameterization of the solution of these conditions).

Note that $X_i^{\tilde{i}}$ can be considered as a 2×2 matrix, and hence $\det X$ is a gauge and flavor symmetry invariant of mass dimension 8.

According to [1], a tree-level superpotential $W_{\text{tree}} = \nu Y_1$ lifts all classical flat directions and, together with the non-perturbatively generated superpotential, leads to dynamical supersymmetry breaking at a scale controlled by the coupling ν.

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Here we wish to consider a more general tree-level superpotential given by:

$$W_{\rm tree} = {\rm tr}\,\mu X + \nu^i Y_i\,. \qquad (2.3)$$

This is not a renormalizable superpotential, though the first term can be easily generated by integrating out some massive non chiral matter, as is present in supersymmetric grand unified models.

Let us first briefly consider a generalization of the Konishi anomaly which boils down to the classical equations of motion. For the anomalous one-loop piece of the relation to be trivially zero, one needs the variation of the field to be independent of the field itself. It is straightforward to convince oneself that the only non-trivial variation satisfying the above requirement is the following:

$$\delta \tilde F^{\tilde\imath}_a = \rho^{\tilde\imath ij}\epsilon_{abcde}T^{bc}_i T^{de}_j\,, \qquad \delta T^{ab}_i = 0\,. \qquad (2.4)$$

Acting on the invariants, the variation above gives:

$$\delta X^{\tilde\imath}_i = 0\,, \qquad \delta Y_i = -2\rho^{\tilde\imath jk}\epsilon_{ij}\epsilon_{\tilde\imath\tilde\jmath}X^{\tilde\jmath}_k\,. \qquad (2.5)$$

We have used the fact that there is no gauge invariant built out of 5 Ts, and that the only gauge invariant built out of 1 $\tilde F$ and 3 Ts is in the **2** of $\mathrm{SU}(2)_T$, and thus is X. Using now the tree-level superpotential, we see that in a supersymmetric vacuum we must have:

$$\nu^i X^{\tilde\jmath}_k = 0\,. \qquad (2.6)$$

This means that as soon as the coupling to the Y_i invariants is turned on at tree-level, the only supersymmetric vacua can occur at zero value for the $X^{\tilde\imath}_i$ invariants. This is going to be crucial for the determination of dynamical supersymmetry breaking.

We now turn to determine the "canonical" Konishi anomaly relation, that is the anomaly associated to the currents generating the variations $\delta\tilde F^{\tilde\imath}_a = \xi^{\tilde\imath}_{\tilde\jmath}\tilde F^{\tilde\jmath}_a$ and $\delta T^{ab}_i = \chi^j_i T^{ab}_j$. Assuming the tree-level superpotential (2.3), the following relations hold in a supersymmetric vacuum:

$$(X\mu)^{\tilde\jmath}_{\tilde\imath} + \delta^{\tilde\jmath}_{\tilde\imath}\nu^i Y_i = \delta^{\tilde\jmath}_{\tilde\imath} S\,, \qquad (2.7)$$

$$(\mu X)^i_j + \delta^i_j\,{\rm tr}\,\mu X + \nu^i Y_j = 3\delta^i_j S\,. \qquad (2.8)$$

The classical equations of motion are recovered by setting S to zero.

By taking traces and substituting back into the equations, it is straightforward to show that the above equations are equivalent to:

$$\nu^i Y_j = 0\,, \qquad (\mu X)^i_j = \delta^i_j S\,. \qquad (2.9)$$

Considering first the classical vacua (i.e. $S=0$), we notice that (2.9) together with (2.6) implies that as soon as we turn on the $\nu^i Y_i$ coupling, all the flat directions are lifted since the equations imply that $X^{\tilde\imath}_i = 0 = Y_i$. On the other hand, if $\nu^i = 0$, not all flat directions are lifted, since the expectation values of Y_i remain unconstrained. For instance, take the following values for the matter fields, which satisfy the D-flatness conditions:

$$T^{12}_1 = a\,, \qquad T^{12}_2 = b\,, \qquad \tilde F^{\tilde 1}_1 = \tilde F^{\tilde 2}_2 = \sqrt{|a|^2+|b|^2}\,. \qquad (2.10)$$

For the above values we have that $Y_1, Y_2 \neq 0$ while $X^{\tilde\imath}_i = 0$, and one can also check that the classical equations of motion are satisfied (for $\nu^i = 0$ and $\mu \neq 0$).

2.1 Unbroken supersymmetry

We now solve for (2.9), assuming that $\nu^i = 0$:

$$X_i^{\tilde{i}} = S(\mu^{-1})_i^{\tilde{i}} . \tag{2.11}$$

and use the above result to compute the coupling dependent piece of the effective superpotential by using the effective relations, valid for expectation values in a supersymmetric vacuum:

$$X_i^{\tilde{i}} = \frac{\partial W_{\text{eff}}}{\partial \mu_{\tilde{i}}^i} . \tag{2.12}$$

By integrating the above equation, we find:

$$W_{\text{eff}} = C(S) + S \log(\Lambda^2 \det \mu) , \tag{2.13}$$

where $C(S)$ is a piece of the superpotential which is independent of the couplings $\mu_{\tilde{i}}^i$, and Λ is the holomorphic scale at high energies.

We can now apply the linearity principle [12] and integrate in the effective fields X by subtracting the linear coupling $\operatorname{tr} \mu X$, to obtain:

$$W_{\text{eff}} = C(S) - 2S \left(1 - \log \frac{S}{\Lambda^3}\right) - S \log \frac{\det X}{\Lambda^8} , \tag{2.14}$$

where the procedure is obviously unaffected by the form of $C(S)$.

To find $C(S)$, we use the following argument. We have to match the above result to the expected low-energy superpotential of the theory where now $W_{\text{tree}} = 0$. The invariants are expected to take on generic expectation values, and break entirely the gauge group. We thus expect the following S-dependent Veneziano-Yankielowicz-type [13] superpotential:

$$W_{\text{eff}} = S \left(1 - \log \frac{S}{\Lambda^3}\right) - S \log \frac{\det X}{\Lambda^8} . \tag{2.15}$$

At this point one may question the validity of introducing the gluino condensate S in (2.15) since there is no unbroken low energy gauge group. However, this is a situation similar to SU(N_c) SQCD with $N_f = N_c - 1$ flavors,[1] where a non-perturbative superpotential is still generated by the instantons in the broken gauge group, and correspondingly one can introduce a glueball superfield S and its related Veneziano-Yankielowicz superpotential as explained for instance in [14] (see also [4]).

By matching with the above, we find:

$$C(S) = 3S \left(1 - \log \frac{S}{\Lambda^3}\right) . \tag{2.16}$$

We can thus write the complete effective superpotential (2.13) as:

$$W_{\text{eff}} = 3S \left(1 - \log \frac{S}{\Lambda^3}\right) + S \log \det(\Lambda^2 \mu). \tag{2.17}$$

We will see that the coefficient of 3 is the number of vacua of the theory with $W_{\text{tree}} = \operatorname{tr} \mu X$.

[1] Indeed, the total matter index, which plays the role of N_f in our case, is $n = 2 \cdot \frac{1}{2} + 2 \cdot \frac{3}{2} = 4 = 5 - 1$.

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Minimizing (2.15) with respect to S, we get that:

$$S = \frac{\Lambda^{11}}{\det X}\,, \tag{2.18}$$

The non-perturbatively generated superpotential is thus:

$$W_{np} = \frac{\Lambda^{11}}{\det X}\,. \tag{2.19}$$

This is nothing else than the superpotential determined by Affleck, Dine and Seiberg [1] using symmetry arguments. Recall that it is generated by a one-instanton contribution (indeed $3N - n = 11$) as it should be in a theory with $N - n = 1$.

Now that we have derived W_{np} we can check the vacuum structure of:

$$W_{\text{eff}} = W_{\text{tree}} + W_{np} = \operatorname{tr}\mu X + \frac{\Lambda^{11}}{\det X}\,. \tag{2.20}$$

Extremizing the above with respect to X, we find:

$$W_{\text{eff}}|_{extr} = 3\frac{\Lambda^{11}}{\det X}\,, \qquad \text{with} \qquad \left(\frac{\det X}{\Lambda^8}\right)^3 = \frac{1}{\Lambda^2 \det \mu}\,. \tag{2.21}$$

Note that the constraint on $\det X$ is cubic, resulting in 3 vacua. Of course since there is no coupling for the Y_is, all these vacua are additionally labeled by two flat directions.

Another way to determine that there must be 3 vacua for $\nu^i = 0$ is the following. When $W_{\text{tree}} = \operatorname{tr}\mu X$ the generic situation is that Y_i have non trivial expectation values. Then classically we can take for instance a configuration like (2.10). These background matter fields break the gauge group from SU(5) to SU(3). Out of the 30 matter superfields, 16 are eaten by the gauge bosons which become massive, while two remain massless but are neutral, since they parameterize the flat directions. The remaining 12 matter fields fit into 2 fundamental and 2 anti-fundamental representations of SU(3). Thus the effective gauge theory is SU(3) SQCD with $N_f = 2$. Moreover W_{tree} gives a tree level mass to the quarks, so that the low-energy theory is pure SU(3) SYM, which has 3 supersymmetric vacua. This is also an additional justification for the Veneziano-Yankielowicz piece in (2.15).

Let us now briefly comment on a slightly different route, inspired by [6]. We could add to the tree level superpotential a piece like $W_{\text{tree}} = \cdots + \lambda \det X$, so that the classical equations of motion would allow for a (frozen) expectation value for X, though only when $\nu^i = 0$. The solution to the Konishi anomaly equations including the above higher order term would consist of two branches, one which classically reduces to the origin and the other to the (possibly large) expectation value. This way one forces the theory to be in a specific vacuum where the gauge group is broken and a low-energy superpotential like (2.15) is expected. Then by analytic continuation one recovers (2.17), with additional terms depending on λ. The vacuum structure of such a theory consists of 4 vacua, the 3 which exist for $\lambda = 0$ and the additional one with classical broken gauge symmetry, which is pushed to infinity in the vanishing λ limit.

2.2 Dynamical supersymmetry breaking

Let us now finally come to supersymmetry breaking. To recapitulate the situation for $\nu^i = 0$, we have seen that it is quite similar to SQCD with $N_f < N_c$: For $W_{\rm tree} = 0$, the non-perturbative superpotential is of runaway behavior (though in this case (2.19) is still flat in the Y^i directions), thus pushing all supersymmetric vacua to infinity. When $W_{\rm tree} = {\rm tr}\,\mu X$, we recover 3 supersymmetric vacua, all being parameterized by 2 additional flat directions.

Now turning on the coupling $W_{\rm tree} = \nu^i Y_i + \cdots$ we see that it immediately leads to supersymmetry breaking: From the classical variation (2.6), we learn that X must vanish. But from the reasoning of the previous subsection, we had learned that W_{np} is precisely singular at the origin. Now, by virtue of the linearity principle W_{np} must be independent of the couplings in $W_{\rm tree}$, and is thus exact also for $\nu^i \neq 0$. The outcome is that $W_{\rm eff}$ has no extremum, and thus there is no supersymmetric vacuum in the theory. However, since the $\nu^i Y_i$ term lifts all flat directions, while W_{np} is singular at the origin, the potential must have a minimum at a finite value of the fields, which will be the non-supersymmetric vacuum of the theory [1].

Note in this respect that if $W_{\rm tree} = {\rm tr}\,\mu X + \nu^i Y_i$, and the ν^is are kept small, we expect three minima of the potential close to the original values of the 3 supersymmetric vacua (without flat directions of course), with the supersymmetry breaking presumably lifting the degeneracy among them. On the other hand, if a $\lambda \det X$ coupling is added, we expect the 4th vacuum to be lifted at a much higher energy than the others since the $X \neq 0$ vacuum is already ruled out classically.

We have thus established dynamical supersymmetry breaking by deriving through the Konishi anomaly first the non perturbative superpotential and then checking that an additional coupling leads to a total effective superpotential with no supersymmetric vacua.

An alternative way to show supersymmetry breaking still exploiting the Konishi anomaly would have been to use (2.6) and (2.9) to show that $Y_i = X_i^{\tilde{i}} = S = 0$ in a supersymmetric vacuum. Then one could have argued as in [1] that in such a vacuum the global U(1) symmetries are unbroken, and thus the effective fields must satisfy the 't Hooft anomaly matching conditions, which is possible only at the price of an extremely odd effective field content. Or in the spirit of [2] one could have computed a one-instanton contribution to a correlator involving the above invariants, and upon finding a non-zero result the conclusion is that, for instance, $S \neq 0$, which implies that supersymmetry is broken.

3. SU(N) chiral gauge theories with one family

In this section we will consider $\mathcal{N} = 1$ supersymmetric gauge theories with gauge group SU(N) ($N \geq 5$) with one matter field in the antisymmetric representation T^{ab} and $N-4$ matter fields in the anti-fundamental representation $\tilde{F}^i_a$, $i = 1, \ldots, N-4$. The matter content is such that the gauge anomaly cancels.

For all $N \geq 6$, we have the following type of invariants:

$$Y^{ij} = \tilde{F}^i_a \tilde{F}^j_b T^{ab}\,, \qquad (3.1)$$

which are in the antisymmetric representation of the flavor group $\mathrm{SU}(N-4)$. For N even, we have an additional invariant which is the pfaffian of the antisymmetric tensor:

$$Z = \epsilon_{a_1 \ldots a_N} T^{a_1 a_2} \ldots T^{a_{N-1} a_N} = \mathrm{Pf}\, T\,. \tag{3.2}$$

Note that no invariant at all can be written for the SU(5) theory with one family, which thus does not have any flat direction nor the possibility to have a tree-level superpotential.

The simplest tree-level superpotential is thus:

$$W_{\text{tree}} = \nu_{ij} Y^{ij} + \lambda Z\,. \tag{3.3}$$

Note that the second term is present only for N even, and is non-renormalizable for $N > 6$.

We can now write the Konishi anomaly relations, for the expectation values in a supersymmetric vacuum:

$$2\nu_{ik} Y^{kj} = \delta_i^j S\,, \tag{3.4}$$

$$\nu_{ij} Y^{ij} + \frac{N}{2}\lambda Z = (N-2) S\,, \tag{3.5}$$

the second term on the left hand side of (3.5) being present only if N is even.

3.1 Odd N

We immediately see that if N is odd, the only solution is $Y^{ij} = S = 0$ (in the SU(5) case, the only solution is $S = 0$ since also the first term of W_{tree} cannot be written).

In this case, we do not dispose of the additional X-type invariants as in the situation with two families, so that we cannot argue as in the previous section. However the situation is also clearly different: for instance in the SU(5) case, no invariant can be written and hence no W_{np} either can be written. We have thus to resort to the arguments of [1, 2] and say that the conditions $Y^{ij} = S = 0$ are not consistent either with a credible low-energy spectrum, or with one-instanton calculations.

At this point we comment on the chiral ring of these theories. It turns out that its structure is very simple (somewhat like in SQCD), as it is generated only by the two invariants:

$$S = -\frac{1}{32\pi^2} \mathrm{tr} W^\alpha W_\alpha\,, \quad \text{and} \quad Y^{ij} = \tilde{F}_a^i \tilde{F}_b^j T^{ab}\,, \tag{3.6}$$

(the second invariant being of course zero for SU(5).)

To see this, consider the chiral ring relations:

$$\begin{aligned} W_{\alpha c}^a W_{\beta b}^c &= -W_{\beta c}^a W_{\alpha b}^c\,, \\ W_{\alpha a}^b \tilde{F}_b^i &= 0\,, \\ W_{\alpha c}^a T^{cb} &= W_{\alpha c}^b T^{ca}\,. \end{aligned} \tag{3.7}$$

We can construct singlets using the primitive invariants

$$\delta_b^a, \quad \epsilon^{a_1 \ldots a_N} \quad \text{and} \quad \epsilon_{a_1 \ldots a_N}\,. \tag{3.8}$$

Let us first look at all the tensor structures that can be constructed with δ^a_b only, up to the relations (3.7). They are, in addition to the fundamental fields and the two invariants (3.6):

$$\begin{aligned} F^{ai} &\equiv \tilde{F}^i_b T^{ab}\,, \\ S^a_b &\equiv \epsilon^{\alpha\beta} W^a_{\alpha c} W^c_{\beta b}\,, \\ T^{ab}_\alpha &\equiv W^a_{\alpha c} T^{cb} = T^{ba}_\alpha\,, \\ \Sigma^{ab} &\equiv \epsilon^{\alpha\beta} W^a_{\alpha c} W^c_{\beta d} T^{db} = -\Sigma^{ba}\,. \end{aligned} \tag{3.9}$$

It is easy to convince oneself employing (3.7) that no more independent structures can be constructed by simple contraction of the indices and also that the tensors (3.9) have the indicated symmetry properties. Gauge singlets must then be constructed by contracting the above with the ϵ-tensors. Since $\epsilon_{a_1\ldots a_N}\epsilon^{b_1\ldots b_N}$ is a product of δ's, one can use either all ϵ in the fundamental or all ϵ in the antifundamental. Using $\epsilon^{a_1\cdots a_N}$ will not work because of the "lack of indices" (recall that $i = 1\ldots N-4$) so the only possibility is to use $\epsilon_{a_1\ldots a_N}$ which restricts the possible tensors that can be used to the two singlets in (3.6) and F^{ai}, T^{ab} (one of the fundamental fields), T^{ab}_α and Σ^{ab}.

Now recall that T^{ab} has one zero eigenvalue, so that by an SU(N) transformation we can take $T^{aN} = 0$. We then use the chiral ring relations to show that also

$$F^{Ni} = T^{aN}_\alpha = \Sigma^{aN} = 0\,. \tag{3.10}$$

Thus no further invariants can be constructed with the ϵ-tensor which always carries a $a = N$ index and we are left with (3.6) as the only possibility. This shows that no non-trivial generalized Konishi anomaly relations can be constructed for these theories.

It would be interesting to study the chiral rings for N even and for theories with more families. Though the above argument is not applicable, we believe that their structure is the same, i.e. the only generators are the glueball superfield and the basic invariants.

3.2 Even N

On the other hand, when N is even and $\lambda \neq 0$, the Konishi relations (3.4), (3.5) can be solved to give:

$$Y^{ij} = \frac{1}{2} S(\nu^{-1})^{ij}\,, \qquad Z = \frac{S}{\lambda}\,. \tag{3.11}$$

This case can be solved as in the previous section (see also [6] for SU(6)), finding the expected non-perturbative superpotential. In this case however, we have already all the invariants in $W_{\rm tree}$, and we can check that there is no additional relation like (2.6). Hence, there is no contradiction and all the supersymmetric vacua found in this way are not lifted, supersymmetry being unbroken.

Let us quickly review how to derive W_{np} and the number of vacua in this case. From (3.11), we find straightforwardly, for $N = 2k$:

$$W_{\rm eff} = C(S) + S\log(\Lambda^{k-3}\lambda) + \frac{1}{2} S \log\det\nu = C(S) + S\log(\Lambda^{k-3}\lambda\,{\rm Pf}\,\nu)\,. \tag{3.12}$$

We now proceed to integrate in the effective fields Y^{ij} and Z, by subtracting $W_{\rm tree}$ (3.3) and solving (3.11) for the invariants. The result is:

$$W_{\rm eff} = C'(S) - (k-1)S\left(1 - \log\frac{S}{\Lambda^3}\right) - S\log\frac{Z\,{\rm Pf}\,Y}{\Lambda^{4k-6}}\,, \qquad (3.13)$$

where we absorbed in $C'(S)$ a trivial term linear in S. But here we know what the coefficient of the $S\log S$ piece should be: when $W_{\rm tree} = 0$, the invariants take arbitrary values and the gauge group is maximally broken. By analyzing the D-flatness conditions, one finds that the effective theory is always a pure Sp(4) SYM with massless neutral fields parameterizing the flat directions. Hence the overall coefficient of the Veneziano-Yankielowicz term in (3.13) must be 3. In turn this implies that the exact effective superpotential is given by:

$$W_{\rm eff} = (k+2)S\left(1 - \log\frac{S}{\Lambda^3}\right) + S\log(\Lambda^{k-3}\lambda\,{\rm Pf}\,\nu)\,, \qquad (3.14)$$

and that there should be $k+2$ supersymmetric vacua.

This is confirmed if we integrate out S in (3.13), after having substituted for $C'(S)$, to find that:

$$W_{np} = 3\left(\frac{\Lambda^{2N+3}}{Z\,{\rm Pf}\,Y}\right)^{\frac{1}{3}}. \qquad (3.15)$$

Extremizing then $W_{\rm eff} = W_{\rm tree} + W_{np}$ with respect to Y^{ij} and Z one again finds $k+2$ solutions. Of course, if we take $\lambda = 0$, we have a runaway behavior and no stable vacuum.

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